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COMBINED FREE AND FORCED CONVECTION NEAR A VERTICAL WALL DUE TO OSCILLATIONS IN THE WALL VELOCITY AND TEMPERATURE

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NOMENCLATURE

- g, acceleration due to gravity;
- β, coefficient of volume expansion;
- α, thermal diffusivity;
- ν, coefficient of kinematic viscosity;
- $\bar{T}_w, \bar{T}_\infty$, constant mean wall temperature, constant free stream temperature;
- U_0, mean velocity of wall oscillations;
- ε, a very small number;
- δ, dimensionless boundary-layer thickness;
- Pr, Prandtl number;
- x, y, coordinates in x, y directions;
- η, non-dimensional perpendicular distance from the wall;
- U, V, T, t, ω, fluid velocities in x, y directions, fluid temperature, time, frequency of oscillations in their non-dimensional forms. Barred quantities denote their dimensional forms.

Subscripts

- s, steady part;
- $1, 2$, oscillating component, out of phase component;
- x, y, t, partial differentiation with respect to these variables.

1. INTRODUCTION

STUDY of combined free and forced convection near a wall whose temperature and velocity oscillate about a mean is important from a practical point of view. Uniform velocity or constant wall temperatures are only ideal cases and in reality are subject to periodic variations occurring at long intervals which is a case of low frequency oscillations or at short intervals corresponding to high frequency oscillations. The variations may not be strictly periodic but may very nearly be so. The vertical motion of a rocket through still air having such approximate small periodic changes in its velocity and wall temperature can be likened to a model of flat plate in motion with small variations (from their constant values) in its velocity and temperature. Study of high frequency oscillations in heat transfer near a wall which might be in periodically varying relative motion is of some consequence in the working of liquid rocket and turbojet engines.

Nanda and Sharma [1] studied the free convection in the boundary layer near a vertical wall with its temperature oscillating about a non-zero mean. We have extended their study by imposing a motion on the flat wall varying periodically about a steady mean. Though the manner of analysis follows the lines of Nanda and Sharma [1], some new results evidently reflecting the effects of the imposed oscillatory motion have been obtained and are expected to be of practical interest in problems of the nature quoted above. It is of special interest to note that in case of high frequency oscillations there always exists a fluid layer parallel to the wall at a distance $\eta = 0.37$ which travels with a velocity oscillating

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in phase with the wall velocity for all values of the Grashof number.

2. FORMULATION OF THE PROBLEM AND BASIC EQUATION

We consider an infinite vertical flat plate whose temperature and velocity oscillate in time about non-zero means. Assuming free stream temperature T_∞ as constant, the vertical direction of the plate as positive x-axis, and y-axis perpendicular to the plate, the boundary-layer equations of an incompressible viscous fluid are

$$\bar{U}_{\bar{t}} + \bar{U}\bar{U}_{\bar{x}} + \bar{V}\bar{U}_{\bar{y}} = g\beta(\bar{T} - \bar{T}_\infty) + \nu \bar{U}_{\bar{y}\bar{y}} \tag{1}$$

$$\bar{U}_{\bar{x}} + \bar{V}_{\bar{y}} = 0 \tag{2}$$

$$\bar{T}_{\bar{t}} + \bar{U}\bar{T}_{\bar{x}} + \bar{V}\bar{T}_{\bar{y}} = \alpha \bar{T}_{\bar{y}\bar{y}}. \tag{3}$$

The boundary conditions are

$$\left.\begin{aligned} \bar{y} = 0; \quad & \bar{U} = U_0(1 + \varepsilon \cos \bar{\omega}\bar{t}) \\ & \bar{V} = 0 \\ & \bar{T} - \bar{T}_\infty = (\bar{T}_w - \bar{T}_\infty)(1 + \varepsilon \cos \bar{\omega}\bar{t}) \\ \bar{y} \to \infty; \quad & \bar{U}, \bar{V} \to 0; \quad \bar{T} \to \bar{T}_\infty. \end{aligned}\right\} \tag{4}$$

3. METHOD OF SOLUTION OF THE EQUATIONS

Equations (1), (2) and (3) are made non-dimensional by substituting

$$\left.\begin{aligned} U = \frac{\bar{U}}{U_0}, \quad V = \frac{\bar{V}}{U_0}, \quad t = \frac{U_0^2 \bar{t}}{\nu} \\ x = \frac{\bar{x}U_0}{\nu}, \quad y = \bar{y}\frac{U_0}{\nu} \quad \text{and } \theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty} \end{aligned}\right\} \tag{5}$$

and the boundary conditions are

$$\left.\begin{aligned} y = 0; \quad & U = (1 + \varepsilon \cos \omega t) \\ & V = 0 \\ & \theta = (1 + \varepsilon \cos \omega t) \\ y \to \infty; \quad & U, V, \theta \to 0 \end{aligned}\right\} \tag{6}$$

where $\omega = \nu\bar{\omega}/U_0^2$.

The resulting equations are split into steady and non-steady parts on assuming $fk = fks + \varepsilon fkl\, e^{i\omega t}$ ($k = 1, 2, 3$) when f_1, f_2 and f_3 denote U, V and θ respectively. The part 'fks' is the steady component and 'fkl' is the oscillatory component involving ε. The equations are separately solved for low and high frequency ranges. In low frequency range, the solutions for the basic steady parts are found by the Karman–Pohlhausen method as adopted by Squire [2] and in the high frequency range by direct integration method. In the former case the coefficients of the polynomials are obtained from a pair of linear first order differential equations through a series expansion method. Further details are given in the Appendix below.

4. DISCUSSION OF RESULTS

The following observations are made from the graphs drawn on the basis of computed results.

(1) Fluid velocity at $\eta = 0.37$ fluctuates in phase with the wall velocity. Also for small values of Pr and x, the velocity of the fluid layers nearer the wall than $\eta = 0.37$ oscillate with a phase lead over the wall velocity oscillation while the layers farther beyond $\eta = 0.37$ have a phase lag behind it. Fluid temperature oscillation has a constant phase lead of $\pi/2$ over the wall temperature (Fig. 1).

(2) In shear wave the fluid temperature oscillation has always a phase lead over the wall temperature increasing

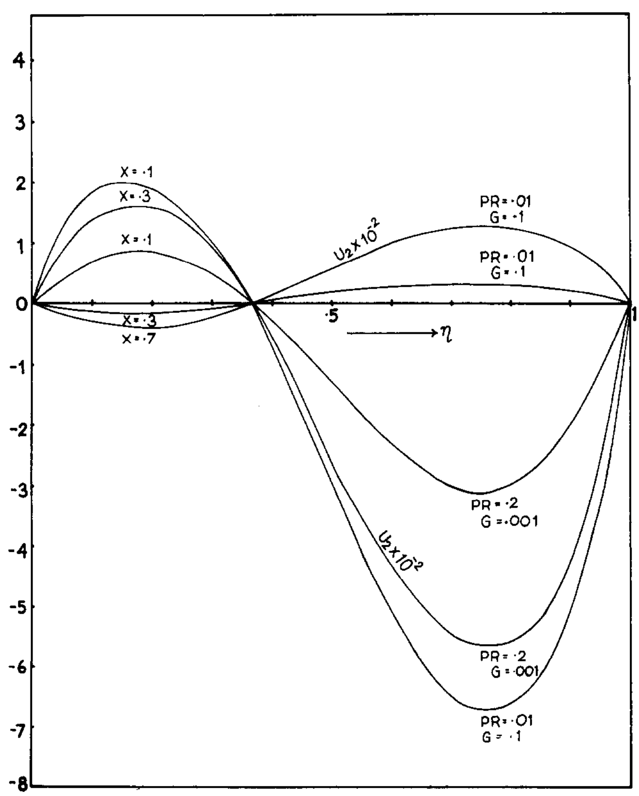

FIG. 1. Velocity distribution in shear wave.

linearly with distance from the wall, the phase difference of the fluid velocity and wall velocity oscillation near the origin fluctuates about a zero mean with decreasing amplitude on the positive side and increasing amplitude on the negative side and with distance from the wall, attaining zero value at the edge of the boundary-layer (Fig. 2).

(3) The amplitudes of oscillation of the velocity and temperature in shear wave actually die down at a distance which is a small fraction of δ. This distance is a measure of the boundary layer which is larger in the thermal case than in the case of velocity oscillations and in either case is entirely contained within the steady boundary-layer as observed earlier (Fig. 3).

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APPENDIX

Equations (1), (2) and (3) on being made non-dimensional with the help of (5) become

$$U_t + UU_x + VU_y = G\theta + U_{yy} \tag{7}$$

$$U_x + V_y = 0 \tag{8}$$

$$\theta_t + U\theta_x + V\theta_y = \frac{1}{Pr}\theta_{yy} \tag{9}$$

where

$$G = \nu g\beta(\bar{T}_w - \bar{T}_\infty)/U_0^3 \quad \text{and} \quad Pr = \nu/\alpha.$$

Introducing the expressions for U, V and θ into (7) and (9) we obtain two sets of equations corresponding to their steady and oscillating components. Our aim is to solve these for low and high frequency ranges.

Low frequency fluctuations

For convenience we split each of U_1, V_1 and θ_1 into in phase and out of phase components as

$$f_1 = fr + if_2 \quad \text{where } f = U, V \text{ or } \theta. \tag{10}$$

Substituting (10) into the above mentioned equations and separating them into real and imaginary parts, we obtain two sets of equations, say (a) and (b) respectively. Let the phase of the velocity and temperature fluctuation at any point within the boundary layer be $\tan^{-1}(U_2/U_r)$ and $\tan^{-1}(\theta_2/\theta_r)$. Since ω is small, the phase shift is small and hence $(-\omega U_2)$ and $(-\omega\theta_2)$ can be neglected from the corresponding equations. Assuming $U = U(T_0, U_0)$, by Taylor's expansion we have $fr = T_0(fs)_{T_0} + U_0(fs)_{U_0}$ where $f = U$, V or θ and the suffix outside the bracket indicates partial differentiation with respect to it. Following the Karman–Pohlhausen method as adopted by Squire [2] we write

$$\left.\begin{aligned} U_2 &= B_1(\eta - 3\eta^3 + 2\eta^4) \\ \theta_2 &= A_1(\eta - 3\eta^3 + 2\eta^4) + \tfrac{1}{2}Pr\omega\delta^2(\eta^2 - 2\eta^3 + \eta^4) \end{aligned}\right\} \tag{12}$$

where $\eta = y/\delta$, δ is the dimensionless boundary layer thickness, A_1 and B_1 are functions of x and further

$$\left.\begin{aligned} U_s &= (1-\eta)^2(1 - M_x\eta) \\ \theta_s &= (1-\eta)^2. \end{aligned}\right\} \tag{13}$$

Solving (9) with the help of (13) we obtain

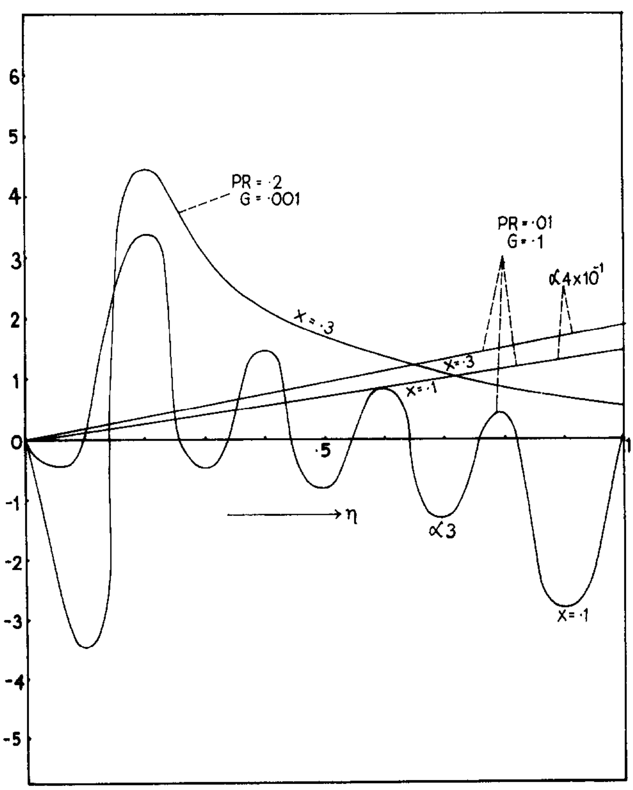

FIG. 2. Phase in shear wave.

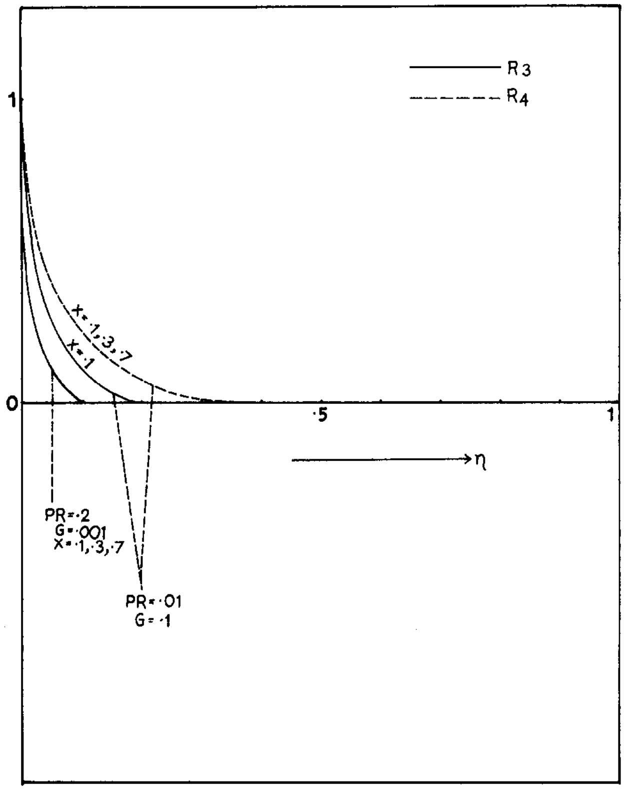

FIG. 3. Amplitude in shear wave.

$$M_x = \{7(4 - 2Pr)/(8 + 7\,Pr)\} - \{\tfrac{1}{3}(4.36)^2(7G/8 + 7\,Pr)^{1/2}\}x^{1/2} \quad (14)$$

and

$$\delta = (4.36)\{(8 + 7\,Pr)/7\,Pr^2\,G\}^{1/2}\,x^{1/4}. \quad (15)$$

From (11) and (15) we obtain

$$\left.\begin{aligned} U_r &= -\tfrac{1}{3}(4.36)^2(7G/8 + Pr)^{1/4}\eta(1 - \eta)^2\,x^{1/2} \\ \theta_r &= 0. \end{aligned}\right\} \quad (16)$$

Integrating the above two sets of equations (a) and (b) with help of (12) and (13), subject to the corresponding boundary conditions, we get a pair of coupled linear first order differential equations which are solved by the series expansion method.

High frequency ranges

The oscillatory boundary-layer thickness $(\nu/\omega)^{1/2}$ for large frequency becomes small and is contained within the steady boundary layer thickness due to mean flow. These expressions can be directly integrated. The longitudinal component of the velocity and temperature of the plate, velocity and temperature in shear wave flow and overall heat transfer have been calculated, but the details are omitted here due to want of space.

Local heat transfer

The quantity of heat transferred from surface to the fluid is given by

$$q = -k(\bar{T}_{\bar{y}})_{\bar{y}=0} = -kU_0\,T_0(\,A_1 \sin\omega t)/\nu\delta.$$

The temperature gradient in shear wave flow is given by

$$Re[\exp(i\omega t)\cdot(\theta_1)_y \quad \text{at } y = 0] = -(\omega Pr)\cos(\omega t + \pi/4).$$